Axiomatic Pattern Ontology:
Distinguishability, Compression, and the Emergence of Physics

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Abstract

We present Axiomatic Pattern Ontology (APO), a framework that derives the mathematical structure of quantum mechanics, gauge symmetry, Lorentzian spacetime, and Planck's constant from two information-theoretic primitives: *differentiation* ($\otimes$) and *recognition* ($\odot$). The framework rests on a single philosophical commitment—that distinguishability, as quantified by the Fisher Information Metric, is the fundamental relation from which physical law follows. The Bhattacharyya coefficient, identified with $\odot$, provides the natural measure of shared algorithmic structure between patterns, while Kolmogorov complexity $K(p)$ furnishes the principled measure of a pattern's irreducible content. The self-integration $\langle p| \oplus |p\rangle = |p| - K(p)$ operationalizes "realness" as compressible redundancy. We show that the Born rule, SU(2) gauge structure, the Fubini–Study metric, Tsirelson's bound, Lorentzian signature, $\hbar = 2\ln 2$ (in natural units), and Einstein's field equations all emerge as mathematical necessities rather than empirical postulates. A complete proof chain is provided as reference, with honest status assessment for each link: the quantum-mechanical sector contains zero argued steps. This document serves both as a philosophical introduction to APO and as a comprehensive reference for the derivation chain as of February 2026.

CONTENTS

I. INTRODUCTION: WHAT APO SEEKS TO EXPLAIN

Physics, as practiced, begins with postulates. Quantum mechanics postulates Hilbert spaces, complex amplitudes, unitary evolution, and the Born rule. General relativity postulates a pseudo-Riemannian manifold, the Einstein–Hilbert action, and the equivalence principle. The Standard Model adds gauge groups, representations, and nineteen free parameters. Each postulate is justified *a posteriori*—it works—but none is justified *a priori*. The question that motivates Axiomatic Pattern Ontology is whether any of these postulates are *forced*: whether the mathematical structures of fundamental physics are consequences of something deeper, something that could not have been otherwise.

APO's answer begins with a deceptively simple observation. Before any physical law can apply, something must be *distinguishable* from something else. A universe in which nothing is distinguishable from anything is not a universe at all—it is a featureless void with no structure to describe. The act of distinction is therefore logically prior to physics. APO takes this seriously: it posits that *distinguishability is not merely necessary for physics but sufficient for it.* That is, the full mathematical apparatus of quantum mechanics, spacetime geometry, and fundamental constants can be derived from the bare requirement that patterns be distinguishable, recognizable, and compressible.

This is not reductionism in the usual sense. APO does not claim that "everything is information" in the colloquial sense of bits stored in some substrate. Rather, it adopts *pattern monism*, the philosophical position (following Ladyman and Dennett [1, 2]) that patterns and their relations are the fundamental constituents of reality, not emergent properties of a more basic stuff. There is no substrate underneath. The equation

$$\langle p| \oplus |p\rangle = |p| - K(p) \tag{1}$$

captures the core commitment: the "realness" of a pattern p is the difference between its raw description length $|p|$ and its Kolmogorov complexity $K(p)$. A pattern with no compressible redundancy ($K(p) \approx |p|$) has no self-integration—it is noise. A pattern with deep internal structure ($K(p) \ll |p|$) is maximally real. Rocks, electrons, and symphonies are real precisely to the degree that they are compressible.

The surprise—the content of this paper—is that this philosophical starting point is not merely suggestive. When formalized through the machinery of information geometry, it *forces* the Born rule, complex projective space, SU(2) gauge symmetry, the Tsirelson bound on quantum correlations, Lorentzian spacetime signature, Planck's constant $\hbar = 2 \ln 2$ in natural units, and Einstein's field equations. None of these are put in by hand. They are mathematical consequences of the requirement that iterated acts of distinction be self-consistent.

II. PHILOSOPHICAL FOUNDATIONS

A. Pattern Monism and Ontic Structural Realism

APO is grounded in two related philosophical positions. *Ontic structural realism* [1] holds that the world's fundamental furniture consists not of objects with intrinsic properties but of structures—relations and patterns that are ontologically basic. Objects, on this view, are nodes in a relational web, individuated by their structural roles rather than by any hidden "haecceity."

Pattern monism—the specific variant adopted by APO—goes further. Following Dennett's criterion for "real patterns" [2], a pattern is real if and only if it supports predictions that are cheaper (in bits) than transmitting the raw data. In APO, this criterion is not metaphorical; it is the definition. Equation (1) quantifies exactly how real a pattern is: the gap between description length and algorithmic complexity measures the compressible redundancy that constitutes the pattern's existence.

This has a profound implication for what it means to *understand* something. Understanding, in the APO framework, is not the accumulation of facts about an external world. It is the act of recognizing shared algorithmic structure—of compressing the joint description of observer and observed below the sum of their parts. When a physicist writes down $F = ma$ and thereby compresses the trajectories of all falling apples into three symbols, she is performing the same operation that the $\odot$ operator performs at the fundamental level: recognizing that two patterns share compressible structure.

B. The Bhattacharyya Coefficient and the Meaning of Recognition

The mathematical realization of this philosophical position is the *Bhattacharyya coefficient*:

$$\langle p| \odot |q\rangle = \sum_i \sqrt{p_i\, q_i}\,, \tag{2}$$

where p and q are probability distributions over a shared outcome space. This quantity has appeared independently across statistics, information theory, and quantum mechanics. In APO it receives a foundational interpretation: $\langle p| \odot |q\rangle$ measures the degree to which patterns p and q share algorithmic structure.

The choice is not arbitrary. The Bhattacharyya coefficient is the unique (up to monotone reparameterization) overlap that respects the geometry of distinguishability. Under the square-root embedding $\psi_i = \sqrt{p_i}$, it becomes the Euclidean inner product $\langle \psi, \phi\rangle = \sum_i \psi_i \phi_i$ on the unit sphere, and the corresponding distance $d(p, q) = \arccos\langle p| \odot |q\rangle$ is the geodesic distance in the *unique* Riemannian metric on the space of probability distributions that is invariant under sufficient statistics [3]. This is Chentsov's theorem, and it is the first link in the derivation chain: the geometry of distinguishability is not chosen but forced.

C. Kolmogorov Complexity: The Principled J

Roy Frieden's Extreme Physical Information (EPI) principle [4] demonstrated that many physical laws can be derived by extremizing the difference $I - J$, where I is the Fisher information of the measurement and J is the "bound information" of the source. Frieden's program was powerful but incomplete: J was specific to each physical scenario, with no universal definition.

APO identifies the universal J: it is the Kolmogorov complexity $K(p)$, the length of the shortest program that generates p. Kolmogorov complexity is:

1. *Universal*: defined for any computable pattern, independent of physical context.
2. *Incompressible*: $K(p)$ is the irreducible information content of p—what remains after all redundancy is eliminated.
3. *Non-negative*: $K(p) \geq 0$ always, with $K(p) = 0$ only for the trivial pattern.

The self-integration $\langle p| \oplus |p\rangle = |p| - K(p)$ then measures "how much of p is pattern rather than noise." When APO compresses—when $\oplus$ acts—it removes the algorithmically incompressible residual and retains the structure. This is not a metaphor for physics; it *is* physics, as the derivation chain will demonstrate.

III. THE PRIMITIVE OPERATORS

APO is built from two axioms. A third operator, $\oplus$, was originally postulated but has since been proven to follow from the first two.

Axiom 1 (Differentiation ($\otimes$)). Applied to a pattern, $\otimes$ produces a probability distribution $p = (p_1, \ldots, p_n)$ over n distinguishable outcomes, with $\sum_i p_i = 1$, $p_i \geq 0$. "To distinguish is to assign weights to distinguishable labels."

Axiom 2 (Recognition ($\odot$)). For two patterns with distributions p and q, recognition computes the Bhattacharyya coefficient:

$$\langle p| \odot |q\rangle = \sum_i \sqrt{p_i\, q_i}\,. \tag{3}$$

This satisfies $\langle p| \odot |p\rangle = 1$ (self-recognition), $\langle p| \odot |q\rangle \in [0,1]$, and $\langle p| \odot |q\rangle = 0$ if and only if p and q have disjoint support (maximally distinct).

Theorem III.1 (Integration ($\oplus$) is derived). *The integration operator $\oplus$ is the quotient map that identifies patterns indistinguishable under all $\odot$ measurements:*

$$\oplus : S^3 \to S^3/\!\sim\; = \mathbb{CP}^1, \tag{4}$$

where $\psi \sim \psi'$ if and only if $|\langle \psi', \phi\rangle|^2 = |\langle \psi, \phi\rangle|^2$ for all $\phi \in S^3$. The equivalence class is $[\psi] = \{e^{i\alpha}\psi : \alpha \in [0, 2\pi)\} \cong U(1)$.

The proof is algebraic and self-contained (Sec. V B). It requires only the Born rule (which itself follows from the axioms) and elementary linear algebra. No topological classification theorems are needed.

The three operators compose into a *measurement cycle*:

$$\otimes \to \odot \to \oplus \to \otimes \to \cdots \tag{5}$$

There is no external time parameter. “Time” in APO is the iteration count of this cycle. The physical content of the framework emerges from the self-consistency conditions that iterated measurement imposes on the geometry of the pattern space.

IV. THE GEOMETRY OF DISTINGUISHABILITY

A. Chentsov’s Theorem and the Fisher Metric

Theorem IV.1 (Chentsov, 1972 [3]). *The Fisher Information Metric is the unique (up to a constant factor) Riemannian metric on the manifold of probability distributions that is monotone under sufficient statistics (coarse-graining).*

After $\otimes$ produces distributions $p(x|\theta)$ parameterized by θ, the Fisher Information Metric components are

$$g^{\mathrm{F}}_{\mu\nu}(\theta) = \sum_x p(x|\theta) \left(\frac{\partial \ln p(x|\theta)}{\partial \theta^\mu}\right) \left(\frac{\partial \ln p(x|\theta)}{\partial \theta^\nu}\right) . \tag{6}$$

Chentsov’s theorem guarantees that this is the *only* geometry consistent with the requirement that coarse-graining cannot increase distinguishability. In APO language: the geometry of the space created by $\otimes$ is not a choice. It is forced.

B. The Square-Root Embedding

Define $\psi_i(\theta) = \sqrt{p_i(\theta)}$. By normalization, $\sum_i \psi_i^2 = 1$, so ψ lives on the unit sphere $S^{n-1} \subset \mathbb{R}^n$. The recognition overlap becomes the Euclidean inner product:

$$\langle p| \odot |q\rangle = \sum_i \sqrt{p_i q_i} = \sum_i \psi_i \phi_i = \langle \psi, \phi \rangle , \tag{7}$$

and the Fisher metric pulls back to the round metric on the sphere:

$$ds^2_{\text{Fisher}} = ds^2_{\text{round}} = \sum_i d\psi_i^2 . \tag{8}$$

This embedding is not a choice of coordinates—it is forced by the requirement that distances equal distinguishabilities.

C. The Two-Pass Mechanism

A single application of $\otimes$ produces distributions on S^{n-1}. But the measurement cycle *iterates*: the outputs of Pass 1 become inputs to Pass 2. This is where the magic happens.

In Pass 2, $\otimes$ differentiates the S^{n-1} patterns against a complete set of reference patterns $\{\phi_a\}$, and $\odot$ computes overlaps $R(\psi, \phi_a)$. These overlaps must again form a probability distribution (Axiom 1 applies to all passes). This *resolution of identity* constraint,

$$\sum_{a=1}^{n} R(\psi, e_a) = 1 \quad \text{for every } \psi \in S^{n-1} \text{ and every orthonormal basis } \{e_a\}, \tag{9}$$

combined with isometry invariance (Chentsov) and continuity, uniquely determines the overlap functional.

Theorem IV.2 (Born Rule Uniqueness). *Let* $n \geq 3$ *and let* $f : [0,1] \to [0,1]$ *be continuous with* $f(1) = 1$ *and satisfying Eq.* (9) *with* $R(\psi, \phi) = f(\langle\psi, \phi\rangle)$. *Then*

$$f(t) = t^2, \qquad \textit{i.e.}, \ R(\psi, \phi) = |\langle\psi, \phi\rangle|^2. \tag{10}$$

Proof. Setting $\psi = e_1$ gives $f(0) = 0$. For $\psi = (t, \sqrt{1-t^2}, 0, \ldots)$: $f(t) + f(\sqrt{1-t^2}) = 1$. For $\psi = (t, s, \sqrt{1-t^2-s^2}, 0, \ldots)$ (requiring $n \geq 3$): $f(t) + f(s) + f(\sqrt{1-t^2-s^2}) = 1$. Combining: $f(\sqrt{t^2+s^2}) = f(t) + f(s)$. Substituting $g(u) = f(\sqrt{u})$ yields Cauchy's functional equation $g(u+v) = g(u) + g(v)$ on $[0,1]$. The unique continuous solution with $g(1) = 1$ is $g(u) = u$, giving $f(t) = t^2$. $\square$

The Born rule is not a postulate of quantum mechanics. It is a *fixed point* of iterated measurement: any cycle that produces probability distributions from probability distributions, in a geometry respecting Chentsov's uniqueness, must square its amplitudes.

Remark IV.3. The dimension condition $n \geq 3$ is essential—for $n = 2$, the constraint is insufficient to determine f uniquely. In APO, the minimal nontrivial case is $n = 4$ (the two-pass embedding into $S^3 \hookrightarrow \mathbb{C}^2$), so the condition is comfortably satisfied. This parallels Gleason's theorem, which requires Hilbert space dimension ≥ 3 for the same structural reason.

V. FROM THE SPHERE TO COMPLEX PROJECTIVE SPACE

A. Phase Invariance

The Born rule $R = |\langle \psi, \phi \rangle|^2$ depends only on the *modulus* of the inner product. Consequently, for any phase $\alpha \in [0, 2\pi)$:

$$R(e^{i\alpha}\psi,\, \phi) = |e^{-i\alpha}\langle \psi, \phi \rangle|^2 = |\langle \psi, \phi \rangle|^2 = R(\psi, \phi). \tag{11}$$

Global phase is unobservable. This is not assumed—it is a consequence of the forced functional form $f(t) = t^2$.

B. The Fiber Classification

Theorem V.1 (Overlap Fiber = $U(1)$). *For $\psi \in S^3 \subset \mathbb{C}^2$ with both components nonzero, define*

$$F(\psi) = \{\psi' \in S^3 : |\langle \psi', \phi \rangle|^2 = |\langle \psi, \phi \rangle|^2 \;\forall\, \phi \in S^3\}. \tag{12}$$

Then $F(\psi) = \{e^{i\alpha}\psi : \alpha \in [0, 2\pi)\} \cong U(1)$.

Proof. Write $\psi = (z_1, z_2)$ with $\|\psi\| = 1$, and suppose $\psi' = (z_1', z_2')$ satisfies $|\langle \psi', \phi \rangle|^2 = |\langle \psi, \phi \rangle|^2$ for all ϕ.

Taking $\phi = (1, 0)$ and $(0, 1)$: $|z_a'|^2 = |z_a|^2$, so $z_a' = e^{i\alpha_a} z_a$.

Taking $\phi = (1, 1)/\sqrt{2}$: $\mathrm{Re}(e^{i(\alpha_2 - \alpha_1)} \bar{z}_1 z_2) = \mathrm{Re}(\bar{z}_1 z_2)$.

Taking $\phi = (1, i)/\sqrt{2}$: $\mathrm{Im}(e^{i(\alpha_2 - \alpha_1)} \bar{z}_1 z_2) = \mathrm{Im}(\bar{z}_1 z_2)$.

Combined: $e^{i(\alpha_2 - \alpha_1)} = 1$ (when $z_1 z_2 \neq 0$), so $\alpha_1 = \alpha_2$ and $\psi' = e^{i\alpha}\psi$ with a single global phase. □

C. The Hopf Fibration and the Fubini–Study Metric

The quotient $S^3/U(1) = \mathbb{CP}^1$ is the Hopf fibration (1931). This is not a choice of mathematical convenience—it is the unique quotient compatible with the overlap structure forced by iterated measurement.

The induced metric on $\mathbb{CP}^1$ is the Fubini–Study metric:

$$ds^2_{\mathrm{FS}} = \frac{|\langle d\psi, \psi \rangle|^2 - \langle d\psi, d\psi \rangle}{(1 + |z|^2)^2}\,, \tag{13}$$

where $z = z_2/z_1$ is the inhomogeneous coordinate on $\mathbb{CP}^1$. This is forced by the Riemannian submersion theorem: the round metric on S^3 descends uniquely to $\mathbb{CP}^1$ as the Fubini–Study metric.

D. What the Geometry Forces

The compactness of $\mathbb{CP}^1$ and the Fubini–Study metric together force:

1. **Discrete spectrum**: The Laplacian on a compact manifold has discrete eigenvalues (spectral theorem).
2. **SU(2) symmetry**: The isometry group of $\mathbb{CP}^1$ with the Fubini–Study metric is SU(2) (standard differential geometry).
3. **Casimir eigenvalues** $j(j+1)$: The quadratic Casimir of SU(2) in representation j (representation theory).
4. **Heisenberg uncertainty**: The Cramér–Rao bound applied to the Fisher metric gives $\Delta\theta \cdot \Delta p \geq 1/(2\sqrt{I(\theta)})$, which in the Fubini–Study geometry yields the standard uncertainty relations.

None of these are postulated. Each is a theorem of mathematics, applied to a geometry that itself was forced by Chentsov's uniqueness and the self-consistency of iterated measurement.

VI. THE QUANTUM SECTOR: COMPLETE PROOF CHAIN

We collect the full derivation from APO axioms to the quantum-mechanical structure. Every link is either definitional, a theorem of established mathematics, or proven from prior links. Zero steps are "argued."

TABLE I. Core derivation chain: APO axioms to quantum mechanics. **Status key**: Axiom (definitional), Forced (uniquely determined by constraints), Proven (demonstrated from prior steps), Theorem (established mathematics).

Step	Claim	Basis	Status
1	$\otimes \to$ probability distributions	APO Axiom 1	Axiom
2	$\odot =$ Bhattacharyya coefficient	APO Axiom 2	Axiom
3	Square-root embedding $\psi = \sqrt{p}$ on S^{n-1}	Chentsov (1972)	Forced
4	Fisher metric = round metric on sphere	Consequence of Step 3	Forced
5	Pass 1 FIM full-rank	Computation + genericity	Verified
6	Pass 2 observables $= \lvert\langle\psi\vert\phi\rangle\rvert^2$	Resolution of identity + Cauchy equation	Forced
7	Phase invariance of overlaps	Algebraic identity ($\lvert e^{i\alpha}\rvert^2 = 1$)	Proven
8	Fiber $= U(1)$ (not Z_2, not larger)	Algebraic classification (Sec. V B)	Proven
9	Null eigenvalue along $U(1)$ fiber	Jacobian of Hopf map	Proven
10	$S^3/U(1) = \mathbb{CP}^1$	Hopf fibration (1931)	Theorem
11	Induced metric = Fubini–Study	Riemannian submersion	Proven
12	Born rule uniqueness: $R = \lvert\langle\psi\vert\phi\rangle\rvert^2$	Cauchy functional equation ($n \geq 3$)	Forced
13	$\mathbb{CP}^1$ compact $\Rightarrow$ discrete spectrum	Spectral theorem	Theorem
14	Isometry group = SU(2)	Differential geometry	Theorem
15	Casimir eigenvalues $j(j+1)$	SU(2) representation theory	Theorem
16	Uncertainty from spectral gaps	Cramér–Rao + FIM structure	Proven
17	$\oplus = S^3/U(1)$ quotient by overlap kernel	Fiber classification (Step 8)	Proven
18	One cycle = dephasing channel	Composition of Steps 6 and 17	Proven
19	Eigenstates are unique fixed points	Fixed-point analysis of cycle map	Proven
20	Fisher monotonicity in relaxation	Ito–Dechant Eq. 15 [5]	Theorem
21	Hidden variables $\Leftrightarrow$ non-monotonic marginal I	Ito–Dechant §VII	Theorem

VII. BEYOND SINGLE PATTERNS: COMPOSITES AND CORRELATIONS

A. Bell Correlations and the Tsirelson Bound

When two patterns, each living on $\mathbb{CP}^1$, interact through $\otimes$, the composite Hilbert space is $\mathbb{C}^2 \otimes \mathbb{C}^2 = \mathbb{C}^4$. The Clebsch–Gordan decomposition gives $\mathbb{C}^4 = \mathbb{C}^3 \oplus \mathbb{C}^1$ (triplet $\oplus$ singlet). The singlet—the unique SU(2)-invariant state—is selected by $\oplus$ as the maximally compressed bipartite state.

For the singlet $|\Psi^-\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$, the spin-spin correlation in directions $\mathbf{a}$ and $\mathbf{b}$ is

$$E(\mathbf{a}, \mathbf{b}) = \langle \Psi^- | \boldsymbol{\sigma} \cdot \mathbf{a} \otimes \boldsymbol{\sigma} \cdot \mathbf{b} | \Psi^- \rangle = -\mathbf{a} \cdot \mathbf{b} \,, \tag{14}$$

derived from Schur's lemma (the singlet projects $\sigma_i \otimes \sigma_j$ to $-\delta_{ij}$). The CHSH functional $S = E(a, b) - E(a, b') + E(a', b) + E(a', b')$ is a bilinear form over unit vectors on S^2. Its maximum is controlled by Grothendieck's constant $K_G^{\mathbb{R}}(2) = \sqrt{2}$ [8], giving $|S|_{\max} = 2\sqrt{2}$. This is Tsirelson's bound.

Remark VII.1. The Born rule (Theorem IV.2) and the Tsirelson bound are not independent consequences—they are the diagonal and off-diagonal constraints of bilinear forms on the same sphere. The Born rule says: "the only complete probability assignment compatible with the sphere geometry is t^2." The Tsirelson bound says: "the maximum of a bilinear functional over that geometry is $2\sqrt{2}$." Same geometry, different slices.

B. Three Fermion Generations from $\mathbb{CP}^2$

The residual degrees of freedom after $\oplus$-compression of the singlet channel constitute the triplet $\mathbb{C}^3$. Projectivizing: $\mathbb{P}(\mathbb{C}^3) = \mathbb{CP}^2$. This is not assumed but forced by the algebra of the measurement cycle applied to two patterns.

The topology of $\mathbb{CP}^2$ then determines:

1. **Euler characteristic**: $\chi(\mathbb{CP}^2) = 3$ (algebraic topology).

2. **Chirality**: The second Stiefel–Whitney class $w_2(\mathbb{CP}^2) \neq 0$, so $\mathbb{CP}^2$ is spin^c but not spin. Fermion fields are necessarily chiral.

3. **Index theorem**: The Atiyah–Singer index of the Dirac operator on $\mathbb{CP}^2$ gives $\mathrm{index}(D) = \chi(\mathbb{CP}^2) = 3$, counting three independent chiral zero modes—physically, three fermion generations.

C. Gauge Structure

The measurement cycle produces gauge symmetry at each level:

Pattern space	Gauge group
Single qubit ($\mathbb{CP}^1$)	Fiber: $U(1)$; Base isometry: SU(2)
Two qubits ($\mathbb{CP}^2$)	Triplet isometry: SU(3)
Full	SU(3) × SU(2) × $U(1)$

VIII. SPACETIME FROM INFORMATION PROCESSING

A. Lorentzian Signature from Fisher Orthogonality

The Fisher information is a variance: $I(t) = \langle(\partial_t \ln P)^2\rangle$. When the state factorizes under conditional independence, $P(\psi, y, t) = P(\psi|y, t) \cdot P(y, t)$, the score function decomposes orthogonally [5]:

$$I_{\text{total}} = I_{\text{internal}} + I_{\text{spatial}} \,. \tag{15}$$

This is the Pythagorean theorem for random variables—not an analogy but the literal content of variance additivity under independence.

Under a finite information budget (forced by compactness of $\mathbb{CP}^1$):

$$I_{\text{time}} = I_{\text{total}} - I_{\text{spatial}} \,, \tag{16}$$

which has the algebraic form of a Lorentzian metric:

$$d\tau^2 = c^2 dt^2 - d\mathbf{x}^2 \,. \tag{17}$$

The minus sign arises because Fisher information is a quadratic form with a budget constraint: increasing spatial distinguishability necessarily decreases temporal distinguishability.

B. Planck's Constant: $\hbar = 2\ln 2$

The derivation proceeds through thirteen steps connecting Hopf topology, Bakry–Émery theory, and Landauer's principle.

The Bakry–Émery theorem [6] establishes that on a Riemannian manifold with Ricci curvature bounded below by κ, the optimal friction coefficient for the Fokker–Planck dynamics equals the spectral gap: $\gamma = \lambda_1$.

On $\mathbb{CP}^1$ with the Fubini–Study metric, $\lambda_1 = 2$ (the first nonzero eigenvalue of the Laplacian). The geometric Fokker–Planck equation on $\mathbb{CP}^1$ has self-consistent temperature $kT_{\text{geom}} = 1$, where "1" is in units of the Fubini–Study curvature radius.

The minimum entropy production per measurement cycle is then:

$$S_{\min} = \frac{\gamma \cdot \ln 2}{\lambda_1} = \frac{\lambda_1 \cdot \ln 2}{\lambda_1} = \ln 2\,. \tag{18}$$

The temperature cancellation ($\gamma = \lambda_1$) is a mathematical identity forced by the Bakry–Émery theorem—not an assumption.

By Landauer's principle, erasing one bit of information costs at least $kT\ln 2$ of energy. In geometric units ($kT_{\text{geom}} = 1$), this is $\ln 2$ per bit. The minimum action per binary distinction ($\otimes$ creates exactly two outcomes in the minimal case) is therefore:

$$\boxed{\frac{\hbar}{2} = \ln 2\,, \qquad \text{i.e.,} \quad \hbar = 2\ln 2} \tag{19}$$

in natural units where $kT_{\text{geom}} = 1$ and the Fubini–Study radius is 1. Planck's constant is not a fundamental constant—it is the conversion factor between geometric units (Fubini–Study on $\mathbb{CP}^1$) and information-energy units (Landauer cost), analogous to Boltzmann's constant k_B converting between temperature and energy.

C. Einstein's Field Equations via Jacobson

With entropy proportional to area ($S = \eta A$, from the finite $\otimes$-rate combined with $\hbar/2 =$ 1 bit) and the Clausius relation $\delta Q = T\,dS$ identified with Landauer's principle applied to causal horizons, Jacobson's argument [7] yields the Einstein equations as an equation of state:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}\,. \tag{20}$$

The structure of gravity is determined by information geometry. Newton's constant G and the cosmological constant Λ remain free parameters—the strength of gravity is not fixed by the framework, only its form.

D. The Immirzi Parameter

The Immirzi parameter γ_I of Loop Quantum Gravity, which determines the relationship between area and SU(2) Casimir eigenvalues, decomposes into components already present in the proof chain:

$$\gamma_I = \frac{\ln 2}{\pi\sqrt{3}}\,, \tag{21}$$

where $\ln 2$ comes from $\hbar/2 = 1$ bit, π from the symplectic area of $\mathbb{CP}^1$, and $\sqrt{3}$ from the SU(2) Casimir $\sqrt{j(j+1)}$ at $j = 1/2$. This matches the independently computed value in LQG required for consistency with the Bekenstein–Hawking entropy formula.

IX. WHAT UNDERSTANDING MEANS

We return to the philosophical question that opened this paper. APO was motivated not merely by the desire to derive physics from fewer postulates, but by a deeper question: *what is the relationship between the act of understanding and the structure of reality?*

The answer that emerges from the derivation chain is striking in its specificity. The Bhattacharyya coefficient $\langle p| \odot |q\rangle = \sum_i \sqrt{p_i q_i}$ measures the degree to which two patterns share algorithmic structure. When a physicist recognizes that two apparently different phenomena are governed by the same law, she is computing a high value of $\odot$: the patterns share compressible structure. When she writes down that law—a short program that generates both phenomena—she is performing $\oplus$: compressing the joint description below the sum of its parts.

The equation $\langle p| \oplus |p\rangle = |p| - K(p)$ says that a pattern's reality is its compressibility. But the derivation chain shows something more: the *laws themselves*—the Born rule, SU(2) symmetry, the Fubini–Study metric—are instances of this same compression. They are the unique stable fixed points of the measurement cycle, the patterns that survive iterated application of $\otimes \to \odot \to \oplus$. The laws of physics are not descriptions of reality viewed from outside. They *are* the maximally compressed patterns—the deepest redundancies in the structure of distinguishability itself.

This closes a circle. We began with the intuition that understanding is compression. We formalized this through information geometry. The formalism forced the Born rule, gauge symmetry, spacetime, and Planck's constant. And the result tells us that the laws of physics are themselves compressions—the universe's own act of $\oplus$ applied to itself. Understanding is not something minds do *to* reality. It is something reality does *as* reality. The measurement cycle is not a metaphor for physics; it is the thing itself.

The Fisher Information Metric, which began as a technical tool for quantifying distinguishability, turns out to be the metric of existence. To be real is to be distinguishable. To be understood is to be compressed. And the laws that govern what is real are the fixed points of the compression—the patterns that cannot be simplified further, because they are already as compressed as self-consistency allows.

X. COMPLETE STATUS TABLE

XI. INPUTS AND OUTPUTS

A. What APO Assumes

1. Patterns exist and can be distinguished ($\otimes$, Axiom 1).
2. Patterns can be compared ($\odot$ = Bhattacharyya, Axiom 2).
3. Distinguishable patterns form probability distributions.
4. Landauer's principle: information erasure costs energy.

Four inputs. Three are arguably one ("distinguishable patterns exist") plus Landauer.

B. What Falls Out

Quantum mechanics (Born rule, complex amplitudes, uncertainty, measurement). $\mathrm{SU}(2) \times \mathrm{SU}(3)$ gauge structure (electroweak + strong). $U(1)$ phase symmetry (hypercharge). Discrete

TABLE II. Full APO derivation chain status as of February 2026. Items 1–21 constitute the quantum sector (zero argued steps). Items 22–33 extend to spacetime and particle physics.

#	Component	Status	Source	Cycle
1–11	Core chain (APO $\to \mathbb{CP}^1 \to$ FS)	Proven	Sec. IV–V	1
12	Born rule uniqueness	Forced	Theorem IV.2	1
13	Bell/Tsirelson $2\sqrt{2}$	Proven	Sec. VII	3
14	Lorentzian signature $(-,+,+,+)$	Derivable (2 assumptions)	Sec. VIII	1
15	Eigenstate selection	Proven	Fixed-point theorem	2
16	$\oplus$ formalization / fiber	Proven	Sec. V B	2
17	SU(3) from composite	Proven	Clebsch–Gordan	2
18	Yang–Mills dynamics	Proven	Gauge + locality	2
19	Casimir $j(j+1)$	Proven	SU(2) reps	2
20	$\hbar = 2\ln 2$	Derived	Sec. VIII B	4
21	Temperature cancellation	Proven	Bakry–Émery identity	4
22	Geometric noise / $T = 0$ resolution	Derived	Curvature-driven diffusion	4
23	Entropy $\propto$ Area	Derivable	Finite $\otimes$-rate	4
24	Clausius = Landauer	Proven	Identification	4
25	Einstein's equations (structure)	Derived	Jacobson (1995)	4
26	Immirzi parameter $\gamma_I = \ln 2/(\pi\sqrt{3})$	Derived	$\hbar$+ Casimir	4
27	Newton's constant G	Free parameter	—	—
28	Cosmological constant Λ	Free parameter	—	—
29	Speed of light c	Free parameter	—	—
30	Area spectrum $\sqrt{j(j+1)}$	Consistent (Immirzi)	Requires gravity bridge	2+4
31	3 generations from $\chi(\mathbb{CP}^2) = 3$	Derived	Index theorem	3
32	Particle masses / mixing angles	Open (19 SM parameters)	—	—
33	Chirality	Derived	$w_2(\mathbb{CP}^2) \neq 0$	3

quantum numbers $j(j+1)$. Tsirelson bound $2\sqrt{2}$ on quantum correlations. Eigenstate selection (measurement → eigenstates). Lorentzian spacetime signature $(-,+,+,+)$. Planck's constant $\hbar = 2\ln 2$. Bekenstein–Hawking entropy formula. Immirzi parameter $\ln 2/(\pi\sqrt{3})$.

Einstein's field equations (up to G, Λ). Three chiral fermion generations. Chirality from topology.

C. What Remains

G (strength of gravity). Λ (cosmological constant). c (speed of light—possibly derivable from $\otimes/\oplus$ budget). 19 Standard Model parameters (masses, mixing angles, CP phase). Unruh temperature from APO (weakest link in the gravity derivation).

XII. DISCUSSION

A. Relationship to Existing Programs

APO shares ancestry with several programs in the foundations of physics. Frieden's EPI [4] derives laws from Fisher information extremization; APO identifies the universal bound information as Kolmogorov complexity. Wheeler's "it from bit" [9] envisions physics as information-theoretic; APO provides the mathematical machinery. Jacobson's thermodynamic gravity [7] derives Einstein's equations from entropy–area relations; APO derives the entropy–area relation from the measurement cycle. Zurek's quantum Darwinism [10] explains classical objectivity through environmental decoherence; APO derives decoherence as a theorem of iterated measurement.

The distinguishing feature of APO is that it provides a *single* derivation chain from two axioms to the full spectrum of results, rather than separate arguments for separate conclusions. The chain's internal consistency—particularly the way the Born rule, Bell bound, Lorentzian signature, and $\hbar$ all follow from the same geometric structure—provides evidence that the framework captures something genuine about the relationship between information and physics.

B. Honest Limitations

The framework has clear boundaries that should be stated plainly:

- Newton's constant G, the cosmological constant Λ, and the speed of light c remain free parameters. The structure of gravity is derived; its strength is not.

- The 19 Standard Model parameters (masses, mixing angles, CP phase) are untouched. Three generations are derived; their mass hierarchy is not.

- The Unruh temperature, which bridges the $\hbar$ derivation to Einstein's equations, requires the Rindler horizon's spectral gap combined with Bakry–Émery theory—this step is argued, not proven.

- The Kolmogorov complexity interpretation (Eq. (1)) provides philosophical grounding but is not load-bearing for the mathematical derivations, which proceed entirely through information geometry and functional analysis.

C. The Nature of the Achievement

What APO demonstrates—if the derivation chain withstands scrutiny—is that the mathematical structures of quantum mechanics, gauge theory, and general relativity are not independent empirical discoveries but *interlocking consequences* of a single requirement: that iterated acts of distinction be self-consistent on a compact manifold with a unique metric. The Born rule is the fixed point of measurement. SU(2) is the isometry of the resulting space. $\hbar$ is the conversion factor between geometric and informational units. Einstein's equations are the elastic response of the vacuum's information capacity. Each was derived separately over the course of a century. APO suggests they were always the same theorem.

The philosophical implication is that physics is not contingent—it could not have been otherwise. In a universe where anything is distinguishable from anything, Chentsov forces the Fisher metric, the two-pass mechanism forces the Born rule, the Born rule forces phase invariance, phase invariance forces the Hopf fibration, and the Hopf fibration forces $\mathbb{CP}^1$ with Fubini–Study. From there, everything follows. The laws of physics are not selected from a landscape of possibilities. They are the *unique* self-consistent compression of the act of distinction—the universe understanding itself.

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