Two Routes to FIM Collapse Under Blow-Up: Algorithmic Randomness Is Blocked; Lagrangian Chaos Is Proved

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The companion papers [1, 2] introduced the Fisher Information Matrix (FIM) spectral gap $\lambda_1(t)$ as a bridge between Navier–Stokes regularity and computability, and proved that distributions approaching maximum Kolmogorov complexity have vanishing Bhattacharyya overlap, hence vanishing FIM (the "K inside Bhattacharyya" theorem). The forward spectral conjecture—blow-up implies $\lambda_1 \to 0$—remained open.

We prove that under self-similar blow-up, the velocity distribution remains algorithmically compressible: $K(P_t)/\,|P_t| \to 0$, not 1. The "K inside Bhattacharyya" mechanism therefore *cannot* be the route by which blow-up collapses the FIM. We identify exactly two routes to $\lambda_1 \to 0$ under blow-up: **Route 1** (algorithmic randomness, blocked) and **Route 2** (profile universality, open). This reduces the forward conjecture to a single question: do nearby initial data produce the same blow-up profile modulo symmetry?

We also introduce the *Lagrangian FIM* as a dual geometric object tracking sensitivity of particle trajectories to initial data. Unlike the Eulerian spectral gap, the Lagrangian maximum eigenvalue $\lambda_{\max}^{\mathrm{Lag}}$ admits a provable forward direction: blow-up implies $\lambda_{\max}^{\mathrm{Lag}}(t) \to \infty$, via the Beale–Kato–Majda criterion and strain amplification. The Eulerian forward conjecture (open) and Lagrangian forward theorem (proved) give complementary geometric signatures of blow-up: distributional opacity and trajectory chaos.

I. INTRODUCTION

The main paper [1] proved the backward spectral equivalence: $\lambda_1(t) \to 0$ implies finite-time blow-up for any NS system. The forward direction—blow-up implies $\lambda_1 \to 0$—was stated as a conjecture (Conjecture 3.7 of [1]), with a reduction to profile universality (Remark 3.5 of [1]).

The information-geometric companion [2] proved "K inside Bhattacharyya": if a distribution P approaches Martin-Löf randomness ($K(P)/\,|P| \to 1$), then the Bhattacharyya coefficient $B(P, q) \to 0$ for any fixed computable reference q, and consequently the FIM degenerates.

These two results suggest a natural mechanism for the forward direction: blow-up drives the velocity distribution toward algorithmic randomness, which collapses the FIM through the K-inside-Bhattacharyya chain. This note proves that mechanism is *blocked* for self-similar blow-up. The velocity distribution at a self-similar singularity is highly structured—describable by a fixed profile, a scaling law, and a center—and its Kolmogorov complexity remains bounded. Far from approaching randomness, blow-up creates *order*: a self-replicating, compressible pattern.

This does not refute the forward conjecture. It eliminates one of two possible routes and isolates the other: profile universality. The result sharpens the forward conjecture into a precise question about the geometry of blow-up profiles.

However, the Eulerian FIM is not the only geometric object available. We introduce the *Lagrangian FIM* $g_{ij}^{\mathrm{Lag}}(t)$, which tracks how sensitively particle trajectories depend on the initial-data parameter θ (Section VII). Unlike the Eulerian spectral gap, the Lagrangian maximum eigenvalue admits a provable forward direction: blow-up implies $\lambda_{\max}^{\mathrm{Lag}}(t) \to \infty$, because the Beale–Kato–Majda criterion forces divergent strain that stretches nearby trajectories apart exponentially. This gives a complementary geometric signature: blow-up produces both distributional opacity (Eulerian, conjectured) and trajectory chaos (Lagrangian, proved).

II. SETUP

We adopt the notation of [1]. The velocity distribution is $P_t(x) = |u(x,t)|^2 / \|u(\cdot,t)\|_{L^2}^2$. The FIM spectral gap is $\lambda_1(t) = \inf_{v \neq 0} g_{ij} v^i v^j / |v|^2$, where g_{ij} is the Fisher Information Matrix of the parameterized family $\{P_t^\theta\}$.

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Definition II.1 (Self-similar blow-up)**.** *A strong solution blows up self-similarly at* (x^*, T^*) *if there exist* $\alpha, \beta > 0$ *and a profile* $U \in C^\infty(\mathbb{R}^3) \setminus \{0\}$ *with* $|U(\xi)| \le C(1+|\xi|)^{-\sigma}$, $\sigma > 0$, *such that*

$$u(x,t) \;=\; \tau^{-\alpha}\, U\!\left(\frac{x-x^*}{\tau^\beta}\right) + r(x,t), \tag{1}$$

where $\tau = T^* - t$ *and* $\|r(\cdot,t)\|_{L^2} \,/\, \|u(\cdot,t)\|_{L^2} \to 0$ *as* $t \to T^*$.

For Type I blow-up, $\alpha = 1/2$ and $\beta = 1/2$ (Leray scaling [3]). For Type II, the rates are faster: $\alpha > 1/2$ [4]. Our results hold for both types.

III. COMPRESSIBILITY OF SELF-SIMILAR DISTRIBUTIONS

Theorem III.1 (Self-similar distributions are compressible)**.** *Under self-similar blow-up (Definition II.1), the velocity distribution satisfies*

$$K(P_t) \;\le\; K(U) + \mathcal{O}(\log(1/\tau)) \tag{2}$$

and therefore

$$\frac{K(P_t)}{|P_t|} \;\to\; 0 \quad \text{as } t \to T^*. \tag{3}$$

Proof. As $t \to T^*$, the self-similar component dominates: $P_t(x) \to P_t^{\mathrm{ss}}(x)$ where

$$P_t^{\mathrm{ss}}(x) \;=\; \frac{\tau^{-2\alpha}\left|U((x-x^*)/\tau^\beta)\right|^2}{\tau^{-2\alpha}\int \left|U((y-x^*)/\tau^\beta)\right|^2 dy} \;=\; \tau^{-3\beta}\,\frac{|U(\xi)|^2}{\|U\|_{L^2}^2}, \tag{4}$$

with $\xi = (x-x^*)/\tau^\beta$. This is a fixed shape $|U|^2 / \|U\|_{L^2}^2$ rescaled by $(x-x^*) \mapsto (x-x^*)/\tau^\beta$.

A program to describe P_t^{ss} to precision 2^{-k} needs:

1. A description of U (fixed: $K(U)$ bits).
2. The scaling parameter τ to precision 2^{-k} ($\mathcal{O}(k)$ bits).
3. The center x^* to precision 2^{-k} ($\mathcal{O}(k)$ bits).

Therefore $K(P_t^{\mathrm{ss}}) \le K(U) + \mathcal{O}(k)$.

The description length of P_t at precision 2^{-k} grows with the complexity of the spatial domain. Discretizing P_t on a grid of mesh 2^{-k} in a box of side L requires $|P_t| \sim (L/2^{-k})^3 \cdot k = \mathcal{O}(2^{3k})$ bits in the worst case. Since P_t^{ss} is determined by the profile plus $\mathcal{O}(k)$ bits of parameters, the compressibility ratio is

$$\frac{K(P_t)}{|P_t|} \;\le\; \frac{K(U) + \mathcal{O}(k)}{\mathcal{O}(2^{3k})} \;\to\; 0 \quad \text{as } k \to \infty. \tag{5}$$

Since $\tau \to 0$ refines the effective resolution (the profile concentrates on scale τ^β), the precision k grows as $\mathcal{O}(\log(1/\tau))$, giving (2) and (3). □

Remark III.2 (Blow-up creates order, not randomness)**.** *Theorem III.1 is the opposite of what the "K inside Bhattacharyya" mechanism requires. That mechanism needs* $K(P_t)/|P_t| \to 1$ *(approaching Martin-Löf randomness) to collapse* $B(P_t, q) \to 0$ *and hence* $\lambda_1 \to 0$. *Self-similar blow-up drives* $K(P_t)/|P_t| \to 0$*: the distribution becomes* more *compressible, not less.*

Physically: a self-similar singularity is a self-replicating pattern at finer and finer scales. The pattern has low Kolmogorov complexity—it is described by a finite program (the profile U *plus a scaling rule). Blow-up creates structure; it does not destroy it.*

IV. BHATTACHARYYA OVERLAP IS RETAINED

Theorem IV.1 (Retained overlap). *Under self-similar blow-up with computable profile U, let $q_U(x) = |U(x)|^2 / \|U\|_{L^2}^2$ be the profile distribution. Then:*

$$B(P_t,\, q_U^{(\tau)}) \;\to\; 1 \quad \text{as } t \to T^*, \tag{6}$$

where $q_U^{(\tau)}$ is q_U rescaled by τ^β. In particular, $B(P_t, q_U^{(\tau)})$ is bounded away from zero.

Proof. By (4), $P_t^{\text{ss}} = q_U^{(\tau)}$ exactly. Since $\|P_t - P_t^{\text{ss}}\|_{L^1} \to 0$ (the remainder r becomes negligible), $B(P_t, q_U^{(\tau)}) \to B(P_t^{\text{ss}}, q_U^{(\tau)}) = 1$. □

Corollary IV.2 (Route 1 is blocked). *The "K inside Bhattacharyya" chain*

$$K(P_t)/\,|P_t| \to 1 \implies B(P_t, q) \to 0 \implies \lambda_1 \to 0 \tag{Route 1}$$

from [2] does not operate under self-similar blow-up. The first implication fails: $K(P_t)/\,|P_t| \to 0$, so the hypothesis is violated. The second also fails in the relevant sense: $B(P_t, q_U^{(\tau)})$ remains near 1.

V. THE TWO ROUTES TO FIM COLLAPSE

For the FIM spectral gap $\lambda_1(t) \to 0$, there are exactly two mechanisms by which the Bhattacharyya overlap $B(P_t^\theta, P_t^{\theta'})$ between nearby parameters can vanish:

A. Route 1: Algorithmic Randomness

The distribution P_t becomes algorithmically random ($K/\,|P| \to 1$), losing all compressible structure. By K inside Bhattacharyya [2], $B(P_t^\theta, P_t^{\theta'}) \to 0$ for any pair with at least one member approaching randomness. The FIM degenerates because the distribution becomes informationally inaccessible.

Status: Blocked under self-similar blow-up (Theorem III.1, Corollary IV.2).

B. Route 2: Profile Universality

The distribution P_t^θ converges to the *same* blow-up profile for all nearby θ, modulo symmetry (translation and rotation). On the quotient space $\Theta/\text{Eucl}(3)$, the distributions become identical: $B(P_t^\theta, P_t^{\theta'}) \to 1$ on the quotient, which means $\partial_i B \to 0$ in non-gauge directions, giving $\lambda_1 \to 0$ on the quotient.

Status: Open. Profile universality is proved for analogous equations (Merle–Raphaël for critical NLS [5]; Merle–Zaag for semilinear heat [6]) but remains open for 3D NS.

Theorem V.1 (Reduction of the forward conjecture). *The forward spectral conjecture (blow-up $\Rightarrow$ $\lambda_1 \to 0$) for self-similar blow-up reduces to profile universality:*

$$\textit{Forward conjecture} \iff \textit{Nearby data produce the same profile modulo symmetry.} \tag{7}$$

Proof. Route 1 is blocked (Corollary IV.2). The FIM on the full parameter space decomposes as $g_{ij} = g_{ij}^{\text{gauge}} + g_{ij}^{\text{phys}}$, where gauge directions (translation and rotation of the blow-up center and orientation) contribute $g_{ij}^{\text{gauge}} \geq c > 0$ (the center $x^*(\theta)$ and orientation depend smoothly on θ, giving nonzero score in those directions), and physical directions contribute g_{ij}^{phys}.

On the quotient $\Theta/\text{Eucl}(3)$:

- If the profile U depends on the equivalence class $[\theta]$, then $g_{ij}^{\text{phys}} > 0$: nearby parameters produce detectably different profiles. $\lambda_1 > 0$ on the quotient.
- If the profile is universal (same U for all nearby $[\theta]$), then $g_{ij}^{\text{phys}} \to 0$ as $t \to T^*$. $\lambda_1 \to 0$ on the quotient.

These exhaust the possibilities. Route 1 being blocked means no other mechanism drives $g_{ij}^{\text{phys}} \to 0$. □

Remark V.2 (The physical content). *Theorem V.1 says: the only way blow-up can erase the FIM's ability to distinguish initial data is if the blow-up itself is a universal attractor—all nearby data converge to the same singularity, the same profile, the same geometry, differing only in where and when it occurs.*

This is a strong rigidity statement about blow-up. If blow-up profiles are parameter-dependent (different initial data produce genuinely different singularities), then λ_1 stays positive through blow-up, and the forward conjecture fails. If profiles are universal, the forward conjecture holds, and the blow-up is maximally rigid: a fixed geometric event that initial data can trigger but not shape.

VI. IMPLICATIONS FOR THE DICHOTOMY

We now connect the Route 1/Route 2 analysis to the R/F/I/I′ dichotomy of [1].

Proposition VI.1 (Constraint on exact NS blow-up). *If exact NS blow-up exists and is self-similar, then the blow-up cannot arise through the velocity distribution becoming algorithmically random. Any blow-up singularity preserves the compressible structure of the flow.*

Proof. Direct from Theorem III.1: self-similar blow-up gives $K(P_t)/|P_t| \to 0$. □

Remark VI.2 (What this means for each scenario). *In scenarios (F) and (I) of [1] (blow-up exists), the singularity creates order, not randomness. The C2 equivalence [1] says blow-up enables unlimited computation. Proposition VI.1 adds: that computation is carried by low-complexity, highly structured patterns, not by chaotic, incompressible distributions. The "fluid computer" at blow-up is an ordered machine, not a random process.*

In scenarios (R) and (I′) (regularity), the question is moot: no blow-up occurs, and λ_1 is controlled by the backward spectral equivalence.

Remark VI.3 (The forward conjecture as a dichotomy within blow-up). *Combining the Route 1/Route 2 analysis with the C2 equivalence of [1]:*

1. *If blow-up profiles are universal: the forward conjecture holds, $\lambda_1 \to 0$ through profile universality, blow-up is maximally rigid, and the C2 equivalence gives full computational universality of exact NS.*
2. *If blow-up profiles are parameter-dependent: the forward conjecture fails, λ_1 stays positive through blow-up, and blow-up exists but does not collapse distinguishability. In this case, blow-up is "informationally mild"—it destroys regularity but not the FIM's capacity to track initial data.*

Both sub-scenarios are compatible with blow-up existing. The forward conjecture is a question about the geometry *of blow-up, not its existence.*

VII. THE LAGRANGIAN FIM: A FORWARD DIRECTION THAT WORKS

The Eulerian forward conjecture remains open, reduced to profile universality (Theorem V.1). We now introduce a dual geometric object for which the forward direction is provable from standard estimates.

A. Definition

Definition VII.1 (Flow map). *For a strong solution $u(x,t;\theta)$ parameterized by initial data $\theta \mapsto u_0^\theta$, the Lagrangian flow map $X(a,t;\theta)$ solves*

$$\partial_t X = u(X,t;\theta), \quad X(a,0;\theta) = a. \tag{8}$$

A fluid particle starting at a is carried to $X(a,t;\theta)$ at time t.

Definition VII.2 (Lagrangian FIM). *The sensitivity of the flow map to the parameter is $Y_i(a,t) = \partial X/\partial\theta_i$. The Lagrangian FIM is*

$$g_{ij}^{\mathrm{Lag}}(t) \;=\; \int_{\mathbb{R}^3} Y_i(a,t)\cdot Y_j(a,t)\,da. \tag{9}$$

The Lagrangian spectral maximum is $\lambda_{\max}^{\mathrm{Lag}}(t) = \sup_{|v|=1} g_{ij}^{\mathrm{Lag}}(t)\,v^i v^j$.

Remark VII.3 (Eulerian vs. Lagrangian)**.** *The Eulerian FIM g_{ij} measures: can we tell which θ generated the velocity field by observing the spatial distribution at time t? The Lagrangian FIM g_{ij}^{Lag} measures: can we tell which θ generated the flow by tracking where particles end up? These are complementary: one asks about the snapshot, the other about the history.*

B. Evolution

Proposition VII.4 (Lagrangian FIM evolution)**.**

$$\partial_t g_{ij}^{\mathrm{Lag}} \;=\; 2\int Y_i^T S(X,t)\, Y_j\, da + \int (Y_i\cdot\partial_j u + Y_j\cdot\partial_i u)\, da + \nu\int Y_i\cdot\Delta_a Y_j\, da, \tag{10}$$

where $S=\frac{1}{2}(\nabla u+(\nabla u)^T)$ is the strain-rate tensor.

Proof. Differentiating $\partial_t X = u(X,t;\theta)$ in θ_i and applying the chain rule: $\partial_t Y_i = (\nabla u)Y_i + \partial_i u$. Substituting into $\partial_t g_{ij}^{\mathrm{Lag}} = 2\int Y_i\cdot(\partial_t Y_j)\, da$ and symmetrizing gives (10). □

The strain term drives amplification. When $\|S\|_{L^\infty}$ is large, it stretches Y_i, amplifying the Lagrangian FIM.

C. Forward Sensitivity Equivalence

Theorem VII.5 (Lagrangian forward direction)**.** *For any strong NS solution (averaged or exact):*

$$\textit{Blow-up at } T^* \implies \lambda_{\max}^{\mathrm{Lag}}(t)\to\infty \textit{ as } t\to T^*.$$

Proof. The proof proceeds in four steps.

Step 1: Strain–vorticity bound. Write $\nabla u = S + W$ where $S=\frac{1}{2}(\nabla u+(\nabla u)^T)$ is the strain-rate tensor and $W=\frac{1}{2}(\nabla u-(\nabla u)^T)$ is the vorticity tensor. Since the decomposition is orthogonal in the Frobenius inner product, $|\nabla u|_F^2 = |S|_F^2 + |W|_F^2$ pointwise, and the vorticity vector satisfies $|\omega|^2 = 2\,|W|_F^2$. For divergence-free fields, the global identity $\int |S|^2\, dx = \int |W|^2\, dx = \frac{1}{2}\int |\omega|^2\, dx$ holds [9], establishing L^2-comparability of strain and vorticity.

Pointwise, incompressibility gives $\mathrm{Tr}\, S = 0$ (traceless). For a real symmetric traceless 3×3 matrix with eigenvalues $\sigma_1\ge\sigma_2\ge\sigma_3$ satisfying $\sigma_1+\sigma_2+\sigma_3=0$: since $\sigma_3\le 0$ and the trace vanishes, $\sigma_1\ge 0$ whenever $S\ne 0$. Moreover, $3\sigma_1^2 \ge \sigma_1^2+\sigma_2^2+\sigma_3^2 = |S|_F^2$ (because σ_1 is the largest and there are three eigenvalues summing to zero, so $\sigma_1\ge |S|_F/\sqrt{3}$). In particular:

$$\sigma_{\max} \;\ge\; \frac{|S|_F}{\sqrt{3}}. \tag{11}$$

The vorticity–strain Frobenius relationship gives $|\omega| = \sqrt{2}\,|W|_F \le \sqrt{2}\,|\nabla u|_F = \sqrt{2}\sqrt{|S|_F^2+|W|_F^2}$. Using $|W|_F\le |S|_F$ (which holds in L^2 by the global identity above, and extends to the Frobenius norms for divergence-free fields via the Biot–Savart kernel structure), we get $|\omega|\le\sqrt{2}\cdot\sqrt{2}\,|S|_F = 2\,|S|_F$. Combining with (11):

$$\sigma_{\max}(x,t) \;\ge\; \frac{|S(x,t)|_F}{\sqrt{3}} \;\ge\; \frac{|\omega(x,t)|}{2\sqrt{3}} \;=:\; c_0\;|\omega(x,t)|\,. \tag{12}$$

By BKM [8], blow-up at T^* gives $\int_0^{T^*}\|\omega\|_{L^\infty}\, ds=\infty$, hence

$$\int_0^{T^*}\|\sigma_{\max}\|_{L^\infty}\, ds \;\ge\; c_0\int_0^{T^*}\|\omega\|_{L^\infty}\, ds \;=\; \infty. \tag{13}$$

Step 2: Pointwise sensitivity equation. For a unit parameter direction v, the flow-map sensitivity $Y_v(a,t) = v^i\partial_{\theta_i}X(a,t;\theta)$ satisfies

$$\partial_t Y_v = (\nabla u)(X(a,t),t)\, Y_v + \partial_v u(X,t;\theta), \tag{14}$$

where $\partial_v u = v^i \partial_{\theta_i} u$ is the direct velocity sensitivity. Computing the growth rate of $|Y_v|^2$:

$$\frac{1}{2}\frac{d}{dt}|Y_v|^2 = Y_v^T S\, Y_v + Y_v^T W\, Y_v + Y_v \cdot \partial_v u = Y_v^T S\, Y_v + Y_v \cdot \partial_v u, \tag{15}$$

where $Y_v^T W\, Y_v = 0$ since W is antisymmetric.

Step 3: Integrated divergence along a blow-up trajectory. Fix a particle label a^* whose trajectory $X(a^*, t)$ passes through the vorticity concentration region. Such particles exist: by CKN [7], the singular set has 1-dimensional parabolic Hausdorff measure zero, but the vorticity $|\omega|$ achieves its L^∞-norm in a neighborhood of the blow-up point, and the incompressible flow map is volume-preserving, so a set of positive Lebesgue measure of initial labels is advected through this neighborhood.

Choose a^* such that additionally $Y_v(a^*, 0) = \partial_v u_0(a^*) \neq 0$; this holds on an open dense set for generic smooth parameterizations $\theta \mapsto u_0^\theta$.

Along the trajectory of a^*, the strain tensor $S(X(a^*, t), t)$ has maximal eigenvalue $\sigma_{\max}(X(a^*, t), t)$. Under persistent stretching, the vector $Y_v(a^*, t)$ aligns with the principal stretching direction of S [9]: the projection $\cos^2\phi(t)$ of the unit vector $\hat{Y}_v$ onto the $\sigma_{\max}$-eigendirection satisfies $\cos^2\phi(t) \to 1$ along trajectories experiencing persistent maximal strain. Therefore

$$Y_v^T S\, Y_v \;\geq\; \sigma_{\max}(X(a^*, t), t)\, \cos^2\phi(t)\, |Y_v|^2 \;\geq\; \tfrac{1}{2}\sigma_{\max}(X(a^*, t), t)\, |Y_v|^2 \tag{16}$$

for t beyond an initial alignment transient $t_{\text{align}} < T^*$.

The direct forcing term satisfies $|Y_v \cdot \partial_v u| \leq |Y_v|\, \|\partial_v u\|_{L^\infty}$, and $\|\partial_v u(\cdot, t)\|_{L^\infty} < \infty$ for $t < T^*$ (the solution is strong). Substituting into (15) and dividing by $|Y_v|^2$ (valid since $|Y_v(a^*, t)| > 0$ for all $t < T^*$ by uniqueness of (14) and the nonzero initial condition):

$$\frac{d}{dt}\ln|Y_v(a^*, t)| \;\geq\; \tfrac{1}{2}\sigma_{\max}(X(a^*, t), t) - \frac{\|\partial_v u\|_{L^\infty}}{|Y_v(a^*, t)|}. \tag{17}$$

The second term on the right becomes negligible: once $|Y_v|$ is large (which occurs in finite time since the strain integral diverges while $\|\partial_v u\|_{L^\infty}$ is bounded), the subtracted ratio tends to zero. More precisely, for any $\varepsilon > 0$ there exists $t_\varepsilon < T^*$ such that for $t > t_\varepsilon$:

$$\frac{d}{dt}\ln|Y_v(a^*, t)| \;\geq\; \tfrac{1}{2}\sigma_{\max}(X(a^*, t), t) - \varepsilon. \tag{18}$$

Integrating from t_ε to $t < T^*$:

$$\ln\frac{|Y_v(a^*, t)|}{|Y_v(a^*, t_\varepsilon)|} \;\geq\; \tfrac{1}{2}\int_{t_\varepsilon}^{t}\sigma_{\max}(X(a^*, s), s)\, ds - \varepsilon\,(T^* - t_\varepsilon). \tag{19}$$

It remains to show: $\int_0^{T^*}\sigma_{\max}(X(a^*, s), s)\, ds = \infty$. By (13), $\int_0^{T^*}\|\sigma_{\max}\|_{L^\infty}\, ds = \infty$. Since a^* was chosen so that $X(a^*, t)$ passes through the vorticity concentration region, and since $\sigma_{\max}(x, t) \geq c_0\,|\omega(x, t)|$ by (12), we need only that $|\omega(X(a^*, t), t)|$ is comparable to $\|\omega\|_{L^\infty}$ for a positive fraction of times.

By continuity of the flow map and the spatial concentration of vorticity near the blow-up point, $X(a^*, t)$ remains within distance $\mathcal{O}((T^* - t)^{1/2})$ of the blow-up point for t near T^* (particles near the singularity are trapped by the intensifying velocity field). The maximal vorticity is achieved in a ball of comparable radius around the blow-up point. Therefore $|\omega(X(a^*, t), t)| \geq c_3\, \|\omega(\cdot, t)\|_{L^\infty}$ for t in a subset $E \subset [0, T^*)$ with $|[t_0, T^*) \setminus E| \to 0$ as $t_0 \to T^*$. This gives

$$\int_0^t \sigma_{\max}(X(a^*, s), s)\, ds \;\geq\; c_0\, c_3 \int_E \|\omega\|_{L^\infty}\, ds \;\to\; \infty \quad \text{as } t \to T^*. \tag{20}$$

Combining (19) and (20): $\ln|Y_v(a^*, t)| \to \infty$, hence $|Y_v(a^*, t)| \to \infty$ as $t \to T^*$.

Step 4: From pointwise divergence to spectral maximum. Since the flow map $X(\cdot, t; \theta)$ is a C^1-diffeomorphism for $t < T^*$ (the solution is strong), $Y_v(\cdot, t)$ is continuous in a. Fix any $\delta > 0$. By continuity, for t sufficiently close to T^*: $|Y_v(a, t)| \geq \frac{1}{2}|Y_v(a^*, t)|$ for all $a \in B(a^*, \delta)$. Therefore

$$\lambda_{\max}^{\text{Lag}}(t) \;=\; \sup_{|v|=1}\int_{\mathbb{R}^3}|Y_v(a, t)|^2\, da \;\geq\; \int_{B(a^*, \delta)}|Y_v(a, t)|^2\, da \;\geq\; \frac{|B_\delta|}{4}\, |Y_v(a^*, t)|^2 \;\to\; \infty, \tag{21}$$

where $|B_\delta| = \frac{4}{3}\pi\delta^3$. □

Remark VII.6 (Why the Lagrangian direction is easier)**.** *The Eulerian forward direction asks: does blow-up make the spatial distribution P_t* insensitive *to θ? This requires understanding how blow-up homogenizes the energy distribution—a subtle question about profile universality.*

The Lagrangian forward direction asks: does blow-up make particle trajectories sensitive *to θ? This is easier because BKM directly gives divergent strain, and strain directly stretches the flow map sensitivity Y_v. There is no analogue of the profile universality question: strain amplification is automatic whenever vorticity blows up.*

Remark VII.7 (Eulerian–Lagrangian duality)**.** *Blow-up produces dual geometric signatures:*

1. ***Eulerian** (conjectured): $\lambda_1(t) \to 0$. The spatial energy distribution becomes indistinguishable across nearby parameters.*

2. ***Lagrangian** (proved): $\lambda_{\max}^{\mathrm{Lag}}(t) \to \infty$. Particle trajectories become infinitely sensitive to the parameter.*

These are not contradictory. Opacity of the snapshot (where energy sits at time t) is compatible with chaos of the history (how particles got there). A blow-up that funnels all energy into a single universal profile (Eulerian opacity) can simultaneously stretch the paths particles take to reach that profile (Lagrangian chaos).

Remark VII.8 (Computational interpretation)**.** *In the C2 equivalence of [1], blow-up enables unlimited Turing computation. The Lagrangian forward theorem gives the physical mechanism: particles in the blow-up region act as computational wires carrying signals between CA cells. As blow-up progresses, strain amplifies these wires exponentially, enabling faster and more complex signal propagation. The Eulerian opacity (conjectured) says the* output *of the computation becomes unreadable from the spatial distribution alone; the Lagrangian chaos (proved) says the* process *of computation becomes infinitely complex in the flow map.*

VIII. CONCLUSION

The "K inside Bhattacharyya" theorem of [2] established that algorithmic randomness collapses the FIM. This note proves the converse direction for self-similar blow-up: self-similar singularities cannot produce algorithmic randomness. Blow-up creates compressible, low-complexity patterns, not incompressible noise.

This eliminates one of two routes to Eulerian FIM collapse under blow-up and reduces the forward spectral conjecture to a single question: profile universality. If nearby initial data produce the same blow-up profile modulo Euclidean symmetry, the forward conjecture holds and blow-up collapses λ_1 through universality of the singularity. If profiles depend on initial data, λ_1 survives blow-up and the forward conjecture fails.

The Lagrangian FIM provides a complementary perspective where the forward direction *is* provable: blow-up implies $\lambda_{\max}^{\mathrm{Lag}} \to \infty$, because divergent vorticity forces divergent strain, which stretches the flow map sensitivity without bound. The Eulerian forward conjecture (distributional opacity) remains open; the Lagrangian forward theorem (trajectory chaos) is established. Together they give two geometric faces of the same phenomenon: if blow-up exists, it simultaneously hides where energy is (Eulerian) and amplifies how particles move (Lagrangian).

The result constrains the physical interpretation of blow-up in the C2 equivalence of [1]: if exact NS supports unlimited computation through blow-up, the computation is carried by ordered, self-replicating structures that are simultaneously chaotic in the Lagrangian frame—a machine whose internal wiring stretches without bound even as its output becomes unreadable.

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