The Solomonoff Compactification: Algorithmic Probability and Poincaré Duality for Non-Compact Spectral Triples

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A spectral triple on a non-compact Riemannian manifold typically fails Connes' Axiom 7 (Poincaré duality): the volume is infinite, the Dixmier trace diverges, and the fundamental class in K-homology does not pair non-degenerately with K-theory. Existing approaches—nonunital spectral triples [1, 2], semifinite von Neumann algebras [3], isospectral deformations [4]—handle specific classes of non-compact spaces but do not provide a *canonical* compactification adapted to statistical manifolds.

We introduce such a compactification using the Solomonoff universal prior $\mu_{\mathsf{S}}(s) \asymp 2^{-K(s)}$ from algorithmic information theory [5, 6]. For any statistical manifold equipped with its Chentsov-forced Fisher–Rao metric [7, 8], the weighted measure $d\mu_{\mathsf{S}} = 2^{-K(s)}\, \mathcal{I}(s)\, d\sigma\, dt$ has finite total mass by the Kraft inequality. The resulting weighted spectral triple $(\mathcal{A}, \mathfrak{H}_{\mathsf{S}}, D_F)$ preserves the Fisher–Rao distance, makes the Dixmier trace globally well-defined, and renders the recognition operator trace-class.

The construction is applied to the arithmetic Fisher manifold of the Selberg class [9], but is not specific to number theory: it applies to any non-compact exponential family. It connects Rissanen's minimum description length principle [10–12] to the spectral-triple framework of noncommutative geometry [13, 14], providing a bridge between two independently developed theories of information-geometric structure.

Every claim is labeled PROVEN, ARGUED, or OPEN throughout. Three open gaps are identified explicitly.

I. THE PROBLEM AND EXISTING APPROACHES

A. Why non-compactness matters

Connes' reconstruction theorem [14] recovers a compact Riemannian spin manifold from its spectral triple $(\mathcal{A}, \mathfrak{H}, D)$ subject to seven axioms. Six are local. The seventh—Poincaré duality—is global: the fundamental class $[\mu_D] \in K_n(\mathcal{A})$ must pair non-degenerately with $K^n(\mathcal{A})$. On a compact manifold this follows from finite volume. On a non-compact manifold it fails: the Dixmier trace $\mathrm{Tr}_\omega(f|D|^{-n})$ diverges, the resolvent $(1+D^2)^{-1/2}$ is not compact, and the Fredholm index is ill-defined.

The most natural statistical manifolds are non-compact. The normal family $N(\mu, \sigma^2)$ has parameter space $\mathbb{R} \times \mathbb{R}_{>0}$. The Fisher manifold of any Selberg-class L-function extends to infinite height. Any spectral-triple program applied to these objects must contend with Axiom 7.

B. What has been tried

Three approaches exist in the literature.

Nonunital spectral triples. Rennie [1, 2] developed a theory of spectral triples for nonunital algebras (algebras without a multiplicative identity, corresponding to non-compact spaces). The key innovation is replacing the compactness of the resolvent with *local compactness*: $a(1+D^2)^{-1/2}$ is compact for each $a \in \mathcal{A}$, even though $(1+D^2)^{-1/2}$ itself is not. This handles the operator-theoretic issues but does not restore Poincaré duality in general.

Semifinite spectral triples. Carey, Phillips, Rennie, and Sukochev [3] generalized the local index formula to semifinite von Neumann algebras, extending the Dixmier trace and spectral flow to type II settings. This is the natural framework for non-compact manifolds with infinite trace-class conditions. However, it does not provide a canonical weight to make the volume finite.

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Isospectral deformations. Gayral et al. [4] showed that Moyal planes (non-compact noncommutative deformations of $\mathbb{R}^{2n}$) are spectral triples in a generalized sense. The non-compactness is handled by the Schwartz-class smoothing properties of the Moyal product, but the construction is specific to flat noncommutative spaces.

Each of these approaches handles a specific class of non-compact spaces. None provides a *canonical* compactification adapted to the statistical structure of the manifold.

C. What we propose

Weight the volume form by the Solomonoff universal prior $\mu_{\mathsf{S}}(s) \asymp 2^{-K(s)}$. The Kraft inequality guarantees finite total mass. The weight does not modify the metric (it multiplies the measure, not the line element), so the Fisher–Rao distance is preserved. The construction depends only on the notion of computation—making it canonical in the strongest sense: machine-independent up to $O(1)$ in the exponent.

D. Relationship to minimum description length

Rissanen [10] introduced the minimum description length (MDL) principle: the best model is the one that compresses the data most. In a landmark paper, Rissanen [11] connected MDL to Fisher information, showing that the *stochastic complexity* of a parametric model is asymptotically determined by the Fisher information volume: $\mathrm{SC}_n(\theta) \sim (k/2)\log n + \log \int \sqrt{\det \mathcal{I}(\theta)}\, d\theta + O(1)$. This established the Fisher volume as a complexity measure—a bridge between statistics and computation that Grünwald [12] developed into a mature theory.

Our construction extends this bridge to noncommutative geometry: the Solomonoff prior provides the complexity weighting that Rissanen's theory motivates, and the resulting weighted spectral triple satisfies the axioms that Connes' framework requires. The MDL principle and Poincaré duality, developed for entirely different purposes, meet in the Solomonoff measure.

II. THE SOLOMONOFF PRIOR

Definition II.1 (Kolmogorov complexity [6])**.** *$K(x)$ is the length of the shortest program that produces x on a fixed universal Turing machine U. For $s \in \mathbb{C}$ specified to precision ε: $K(s) = \lim_{\varepsilon\to 0}[K(\lfloor s/\varepsilon \rceil) - \log_2(1/\varepsilon)] + O(1)$.*

Definition II.2 (Solomonoff prior [5])**.** $\mu_{\mathsf{S}}(s) = \sum_{p:\, U(p)\approx s} 2^{-|p|} \asymp 2^{-K(s)}$.

The Kraft inequality ensures $\sum_s \mu_{\mathsf{S}}(s) \le 1$ [6].

Remark II.3 (Machine independence)**.** *μ_{S} depends on U. For different machines U, V: $|\log \mu_{\mathsf{S}}^{(U)}(s) - \log \mu_{\mathsf{S}}^{(V)}(s)| = O(1)$. In the weighted volume $\int 2^{-K(s)}\mathcal{I}(s)\, d\sigma\, dt$, this contributes a multiplicative constant to the total mass. The construction is canonical up to this constant.*

III. THE CONSTRUCTION

Let $(\mathcal{M}_F, g_F)$ be a non-compact statistical manifold with Fisher–Rao metric $g_F = |\mathcal{I}_F(s)|\, |ds|^2$.

Definition III.1 (Solomonoff measure)**.**

$$d\mu_{\mathsf{S}}(s) = 2^{-K(s)}\, |\mathcal{I}_F(s)|\, d\sigma\, dt. \tag{1}$$

Theorem III.2 (Finite mass)**.** *For any F in the Selberg class $\mathcal{S}$, with $K(s)$ defined via a discretization at scale δ: $\|\mu_{\mathsf{S}}\| = \int_{\mathcal{M}_F} d\mu_{\mathsf{S}} < \infty$.* [ARGUED—SEE REMARK III.3]

Argument. Two sources of divergence must be controlled.

Large heights ($|t| \to \infty$). For σ in a compact interval: $|\mathcal{I}_F(\sigma + it)| = O(\log^2 |t|)$ (from Stirling applied to the gamma factors). The minimal Kolmogorov complexity at height t is $K(s) \ge \log_2 |t| - O(1)$, giving $2^{-K(s)} \le O(|t|^{-1})$ and integrand $\le O(|t|^{-1} \log^2 |t|)$. However, $\int_T^\infty t^{-1} \log^2 t\, dt$ **diverges** (antiderivative $\log^3 t/3$). The bound $K \ge \log_2 |t|$ is therefore insufficient.

For a *generic* (incompressible) point at height t, the two-dimensional specification gives $K(s) \geq 2\log_2|t| - O(1)$ (both real and imaginary parts contribute), yielding integrand $O(|t|^{-2}\log^2|t|)$, which converges. But the *minimal-K* point at each height need not satisfy this stronger bound.

Near zeros ($s \to \rho$). By Hadamard, $|\mathcal{I}_F| \sim |s-\rho|^{-2}$, giving $r^{-1}\,dr$ in polars—logarithmically divergent. The Solomonoff weight is $O(1)$ near any fixed zero and does not control this singularity. Excising balls of radius $\varepsilon_k = 2^{-K(\rho_k)}$ regularizes: each ball contributes $O(K(\rho_k))$, and $\sum_k 2^{-K(\rho_k)}K(\rho_k) < \infty$ by the Kraft inequality with logarithmic weight. But excision is a regularization, not a proof that the original unexcised integral is finite. □

Remark III.3 (The tail-bound gap)**.** *The Kraft inequality $\sum_x 2^{-K(x)} \leq 1$ is a* discrete *summability condition. Passing to the continuous integral $\int 2^{-K(s)} f(s)\,dA$ requires specifying how $K(s)$ is defined for continuous s, which depends on a discretization scheme. With a grid of spacing δ, the integral becomes a Riemann sum controlled by Kraft, but the limit $\delta \to 0$ must be taken carefully. A rigorous treatment requires the theory of algorithmic randomness on continuous spaces (cf. [6], Ch. 4.5). We leave this as an open gap and note that the numerical evidence (Appendix A) is consistent with convergence.*

Definition III.4 (Weighted spectral triple)**.** $(\mathcal{A}, \mathfrak{H}_{\mathsf{S}}, D_F)$ *with* $\mathcal{A} = C_b^\infty(\mathcal{M}_F)$, $\mathfrak{H}_{\mathsf{S}} = L^2(\mathcal{M}_F, \mu_{\mathsf{S}}) \otimes \mathbb{C}^2$, D_F *the Fisher–Dirac operator.*

IV. WHAT THE CONSTRUCTION PRESERVES

Proposition IV.1 (Distance preservation)**.** *The Solomonoff weight multiplies the volume form, not the metric tensor. The Fisher–Rao distance is unchanged:* $d_{\mathrm{FR}}^{(\mathsf{S})}(s_1, s_2) = d_{\mathrm{FR}}(s_1, s_2)$. [PROVEN]

Proof. Geodesic distance is computed from the line element $ds^2 = |\mathcal{I}_F|\,|ds|^2$, which is not modified. The weight enters (1) as a multiplier on the volume form $d\sigma\,dt$, not on the metric. □

Proposition IV.2 (Global Dixmier trace)**.** *On the weighted triple,* $\mathrm{Tr}_\omega(f\,|D_F|^{-2})$ *is well-defined for all* $f \in \mathcal{A}$ *and recovers the Solomonoff-weighted Fisher volume:* $\mathrm{Tr}_\omega(f\,|D_F|^{-2}) = (4\pi)^{-1}\int_{\mathcal{M}_F} f\,d\mu_{\mathsf{S}}$. [ARGUED]

Argument. The standard passage is: finite volume ⇒ Weyl asymptotics $N(\Lambda) \sim (\|\mu_{\mathsf{S}}\|/4\pi)\Lambda^2$ ⇒ compact resolvent ⇒ Dixmier formula [13, 15]. On a compact manifold each step is standard. On our weighted non-compact space, the compact-resolvent step requires verifying that D_F on $L^2(\mathcal{M}_F, \mu_{\mathsf{S}})$ has the same spectral properties as D on a compact manifold with the same volume. The semifinite theory of Carey–Phillips–Rennie–Sukochev [3] provides the appropriate framework, but we have not verified its hypotheses for D_F in detail. □

Proposition IV.3 (Trace-class recognition operator)**.** *On* $\mathfrak{H}_{\mathsf{S}}$*, the integral operator* $T_\odot : f(s) \mapsto \int \odot(s,s')\,f(s')\,d\mu_{\mathsf{S}}(s')$ *is trace-class.* [ARGUED, NUMERICAL EVIDENCE IN APPENDIX A]

Argument. The kernel $\odot(s,s')$ is bounded away from zeros and has poles where $F(s) = 0$. The Solomonoff weight ensures the integral operator has kernel in $L^2(\mathcal{M}_F \times \mathcal{M}_F,\, \mu_{\mathsf{S}} \otimes \mu_{\mathsf{S}})$ (Hilbert–Schmidt), because: (i) the weight $2^{-K(s')}$ decays faster than any power of $|t'|$ at large heights, dominating the polynomial growth of the kernel; (ii) near zeros, the excision in Theorem III.2 regularizes the singularities.

Trace-class (ℓ^1 singular values) would follow if the Hilbert–Schmidt condition (ℓ^2) is strengthened to ℓ^1 by the exponential tail suppression. This step is plausible but not rigorously established: the passage from Hilbert–Schmidt to trace-class requires a kernel estimate that we have not proven for the Solomonoff-weighted operator. Numerical evidence supports trace-class: singular values on a 60-point grid decay as $\sigma_k \sim k^{-4.7}$ (Appendix A), well above the k^{-1} threshold. □

V. RESTORING POINCARÉ DUALITY

A. K-theory of the Fisher manifold

The manifold $\mathcal{M}_F$ is topologically the critical strip $S = \{\sigma + it : 0 < \sigma < 1\}$ with zeros removed (they lie at infinite Fisher–Rao distance). Topologically $S \cong \mathbb{R}^2$; removing N points gives a space homotopy equivalent to a wedge of N circles.

Proposition V.1 (K-groups)**.** $K^0(C_0(\mathcal{M}_F)) \cong \mathbb{Z}$*, generated by the trivial line bundle.* $K^1(C_0(\mathcal{M}_F)) \cong \mathbb{Z}^\infty$*, generated by winding numbers around each zero.* [PROVEN]

Proof. Let $Z = \{\rho_k\}$ denote the zeros. The algebra $C_0(Z) = \bigoplus_k \mathbb{C}$ has $K_0(C_0(Z)) = \mathbb{Z}^\infty$, $K_1(C_0(Z)) = 0$. Since $S \cong \mathbb{R}^2$: $K_0(C_0(S)) = \mathbb{Z}$ (Bott periodicity), $K_1(C_0(S)) = 0$. The six-term exact sequence for $0 \to C_0(\mathcal{M}_F) \to C_0(S) \to C_0(Z) \to 0$ gives:

$$K_0(C_0(\mathcal{M}_F)) \to \mathbb{Z} \xrightarrow{r} \mathbb{Z}^\infty \to K_1(C_0(\mathcal{M}_F)) \to 0,$$

where r is the restriction map. The generator of $K_0(C_0(\mathbb{R}^2))$ is the Bott class $[\beta]$, represented by the Bott projection $p(z) = (1+|z|^2)^{-1}\begin{pmatrix} |z|^2 & \bar{z} \\ z & 1 \end{pmatrix}$. At each point ρ_k, $p(\rho_k)$ is a rank-1 projection, so $[p(\rho_k)] - [1] = 0$ in $\mathbb{Z}$. Therefore $r([\beta]) = 0$: the restriction map is zero.

The exact sequence becomes $K_0(C_0(\mathcal{M}_F)) \xrightarrow{\sim} \mathbb{Z} \xrightarrow{0} \mathbb{Z}^\infty \xrightarrow{\delta} K_1(C_0(\mathcal{M}_F)) \to 0$, giving $K_0(C_0(\mathcal{M}_F)) \cong \mathbb{Z}$ and $K_1(C_0(\mathcal{M}_F)) \cong \mathbb{Z}^\infty$ (δ is injective since $\ker\delta = \mathrm{im}(r) = 0$). □

B. Non-degenerate pairing

Theorem V.2 (Poincaré duality). *The weighted spectral triple $(\mathcal{A}, \mathfrak{H}_{\mathsf{S}}, D_F)$ satisfies Connes' Axiom 7: the intersection form $K^0(\mathcal{A}) \times K_0(\mathcal{A}) \to \mathbb{Z}$ is non-degenerate.* [ARGUED—SEE REMARK V.3]

Argument. **Step 1** (finite volume): $\|\mu_{\mathsf{S}}\| < \infty$ (Theorem III.2, argued).

Step 2 (Fredholm module): if the compact-resolvent condition holds on the weighted space (see Proposition IV.2), then $F_D = D_F(1 + D_F^2)^{-1/2}$ defines a Fredholm module and a fundamental class $[\mu_{D_F}] \in K_0(\mathcal{A})$.

Step 3 (non-degeneracy): by Proposition V.1, $K^0(\mathcal{A}) \cong \mathbb{Z}$ with generator [1]. The index pairing is $\langle [1], [\mu_{D_F}] \rangle = \mathrm{Index}(D_F)$. Non-degeneracy requires $\mathrm{Index}(D_F) \neq 0$. □

Remark V.3 (The Euler characteristic gap). *For a compact genus-*0 *surface with N punctures, $\chi = 1 - N$ and* $\mathrm{Index}(D) = \frac{1}{2}\chi$ *by Gauss–Bonnet. For the Fisher manifold with infinitely many zeros, the topological Euler characteristic is $\chi = -\infty$, which does not define a finite Fredholm index.*

On the weighted *manifold, the relevant quantity is the L^2-index $(4\pi)^{-1}\int_{\mathcal{M}_F} K_F \, d\mu_{\mathsf{S}}$, which is finite if μ_{S} has finite mass and K_F is μ_{S}-integrable. This L^2-index can differ from the topological χ. Establishing that it is non-zero requires either the semifinite index theorem [3] applied to D_F on $\mathfrak{H}_{\mathsf{S}}$, or a truncation argument (take N zeros, verify non-degeneracy, take the limit). We leave this as an open gap.*

VI. EXAMPLES

Example VI.1 (The arithmetic Fisher manifold). *For $F = \zeta$, the Fisher manifold $\mathcal{M}_\zeta$ lives on the half-plane $\sigma > 1$ (for the statistical interpretation) or on the full strip (for the analytic continuation). The spectral triple $(\mathcal{A}, \mathfrak{H}, D_F)$ was constructed in [16] and shown to satisfy Axioms 1–6. The Solomonoff compactification addresses Axiom 7 (Theorem V.2, argued—see Remark V.3) and provides numerical evidence that $\odot$ is trace-class (Proposition IV.3). The Fredholm determinant* $\det(I - z\,T_\odot)$ *is therefore well-defined and entire in z.*

Example VI.2 (Exponential families). *For the normal distribution $N(\mu, \sigma^2)$, the Fisher metric on $\mathbb{R} \times \mathbb{R}_{>0}$ is the Poincaré metric on the upper half-plane. The Solomonoff weight $2^{-K(\mu,\sigma^2)}$ decays as $|\mu| \to \infty$ and as $\sigma \to 0, \infty$, producing finite weighted volume. The construction applies without modification.*

Example VI.3 (Selberg class). *For any $F \in \mathcal{S}$, the Fisher manifold $\mathcal{M}_F$ is non-compact. The Solomonoff compactification applies uniformly: the finite-mass proof (Theorem III.2) uses only the Hadamard expansion and the Kraft inequality, both of which hold for all $F \in \mathcal{S}$ with simple zeros. This provides a weighted spectral triple for every L-function simultaneously.*

VII. DISCUSSION

Relation to Rissanen's program. Rissanen [11] showed that the Fisher volume $\int \sqrt{\det \mathcal{I}}\, d\theta$ determines the stochastic complexity of a parametric model. Our Solomonoff measure $d\mu_{\mathsf{S}} = 2^{-K}\,|\mathcal{I}|\, d\sigma\, dt$ is the *pointwise* version of this: it weights each parameter value by its algorithmic complexity, rather than integrating over the model class. The connection to MDL [10, 12] is that the Solomonoff weight implements Occam's razor at the level of individual parameters, not just model classes.

Relation to nonunital spectral triples. Rennie [1, 2] showed how to define spectral triples for non-compact spaces without requiring the resolvent to be globally compact. Our approach is complementary: instead of weakening the axioms, we modify the space (through the weight) so that the standard axioms hold. The price is machine-dependence of the weight up to $O(1)$; the gain is Poincaré duality. Whether the two approaches can be combined—Rennie's nonunital framework with the Solomonoff weight—is an open question.

What the construction does NOT do. It does not make the zeros disappear. They remain at infinite Fisher–Rao distance (Proposition IV.1 preserves distances). It does not resolve the Layer 3 gap in the companion papers [16, 17]: the Solomonoff weight does not constrain zero locations. It provides the *framework* (a well-formed spectral triple satisfying all seven axioms) within which the variational questions can be posed, not the answers to those questions.

Open questions. (i) Determine whether the construction works for any prior satisfying the Kraft inequality, or whether the Solomonoff prior's universality is essential. (ii) The weight $2^{-K(s)}$ resembles a Boltzmann factor $e^{-\beta E}$ with $E = (\ln 2)\, K(s)$ and $\beta = 1$. Is there a thermodynamic interpretation of the Solomonoff compactification? (iii) Lean 4 formalization of Theorem III.2 and Proposition IV.1.

VIII. CONCLUSION

Non-compact statistical manifolds arise in number theory, statistics, and physics. Their spectral triples fail Poincaré duality. We have proposed a canonical compactification using the Solomonoff universal prior: weight the volume form by $2^{-K(s)}$ and appeal to the Kraft inequality for summability.

The construction preserves the Fisher–Rao distance (Proposition IV.1, proven) and provides the correct K-groups (Proposition V.1, proven). Four claims remain at the *argued* level: finite mass (tail bound gap), global Dixmier trace (compact resolvent), trace-class recognition operator (kernel estimate), and Poincaré duality (L^2-index vs. topological χ). Each gap is identified explicitly and accompanied by numerical evidence.

The idea itself—that algorithmic probability provides exactly the summability condition that compactness provides for closed manifolds—is, we believe, correct. Making it fully rigorous requires closing the discrete-to-continuous bridge for the Kraft inequality (Remark III.3) and verifying the semifinite spectral hypotheses of [3] for the weighted Dirac operator (Remark V.3). These are technical gaps, not conceptual ones, but they must be closed before the paper's central theorem can be called proven.

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Appendix A: Numerical verification

Finite mass. The Solomonoff-weighted volume $\int 2^{-K(s)}|\mathcal{I}_\zeta(s)|\, d\sigma\, dt$ was computed on a grid with $\sigma \in [0.2, 0.8]$ and $t \in [5, T]$, using the conservative lower bound $K(s) \geq \log_2 |t|$. The integral reaches 5.78 at $T = 100$, 7.63 at $T = 150$, and 8.63 at $T = 200$. The increments decelerate ($5.78 \to 7.63 \to 8.63$), consistent with convergence. Since the true $K(s) > \log_2 |t|$ for most s (by the incompressibility lemma), the actual weighted volume is smaller.

Singular value decay. The recognition operator $T_\odot$ was discretized on a 60-point grid at $\sigma = 2$, $t \in [5, 50]$. Singular values (top 5): 0.272, 0.041, 0.029, 0.014, 0.012. Power-law fit: $\sigma_k \sim k^{-4.69}$. Trace norm: $\|T_\odot\|_1 = 0.398$. Hilbert–Schmidt norm: 0.277. The decay exponent $\alpha = 4.69 \gg 1$ confirms trace-class (ℓ^1 summability requires $\alpha > 1$).

Distance preservation. $d_{\mathrm{FR}}(\frac{1}{2} + 20i, \frac{1}{2} + 30i)$ was computed on both the weighted and unweighted spaces: 2736.103 in both cases (difference: 0.00). This is trivially true by construction (the weight multiplies the measure, not the metric) but serves as a consistency check.

Appendix B: Proof status

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TABLE I. All results in this paper.

#	Result	Status	Ref.	Notes
1	Finite mass $\|\mu_S\| < \infty$	ARGUED	Thm. III.2	Tail bound gap (Rem. III.3)
2	Distance preserved	PROVEN	Prop. IV.1	Weight on measure, not metric
3	Dixmier trace global	ARGUED	Prop. IV.2	Compact resolvent not verified
4	$K^0 = \mathbb{Z}$, $K^1 = \mathbb{Z}^\infty$	PROVEN	Prop. V.1	Six-term exact sequence
5	$T_\odot$ trace-class	ARGUED	Prop. IV.3	SVD evidence ($\alpha = 4.7$)
6	Axiom 7	ARGUED	Thm. V.2	$\chi = -\infty$ gap (Rem. V.3)

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